REPETENDS.

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In the February number of the *Michigan Teacher* for 1869, there appeared an article, contributed by Mr. William Wiley of Detroit, entitled, "New Theory of Repetends," in which it is shown that the figures of a repetend are easily deduced from the common fraction successively from right to left, instead of from left to right as is done by the ordinary method.

Thinking it deserving of more notice than it received from its publication in that journal, I propose in this article to reproduce, in substance, Mr. Wiley's method, and also demonstrate some of the curious properties of repetends.

First Principle.—In every common fraction which reduces to a pure repetend the unit's figure of the denominator must be either 1, 3, 7 or 9.

Second Principle.—If the numerator of the fraction be unity, the last figure of the repetend is either 9, 3, 7 or 1, respectively, according as the units of the denominator is 1, 3, 7 or 9.

Third Principle.—If $\frac{1}{d}$ be the fraction that reduces to a repetend, q_1 the last figure before it commences to repeat, r_1 the last and r_2 the next to the last remainder, then evidently

$$r_1=1, \text{ and } q_1 = \frac{10\,r_2-1}{d}; \text{ whence } r_2 = \frac{dq_1+1}{10} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Fourth Principle.—If $q_n, q_{n-1} \,.\,.\,.\,.\,.\, q_2$ and q_1 be the digits of the repetend, then is

$$\frac{1}{d} = \frac{10^{n-1}q_n+10^{n-2}q_{n-1}+\ldots 10q_2+q_1+\frac{1}{d}}{10^n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{and } \frac{r_2}{d} = \frac{10^{n-1}\,q_1+10^{n-2}q_n+ \quad . \; . \; . \quad 10q_3+q_2+\frac{r_2}{d}}{10^n} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which we have the same sequence of digits, the last taking the first place and the remaining figures being each removed one place to the right.

For an example, take $\frac{1}{7}$=.142857$\frac{1}{7}$; then $\frac{5}{7}$=.714285$\frac{5}{7}$. This is evident, since $\frac{1}{7}$=.14285$\frac{5}{7}$, and the value of the fraction at the end is five times the part found, $\frac{5}{7}$=.71428$\frac{4}{7}$; whence $\frac{1}{7}$=.1428571428$\frac{4}{7}$=.142857142855$\frac{5}{7}$. We thus see the necessity of the sequence above stated.

When we multiply (2) by r_2, if we rspresent the tens of $r_2\,q_1$, $r_2\,q_2$, &c., by m_1, m_2, &c., we evidently have

$$r_2\,q_1 = 10m_1+q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1')$$

$$r_2\,q_2+m_1 = 10m_2+q_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2')$$

$$r_2\,q_3+m_2 = 10m_3+q_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3')$$

&c., &c.

We here have the key to Mr. Wiley's method; for by the *second principle* q_1 is known at a glance, and by the third principle r_2 is found by equation (1); whence q_2 becomes known from (1′), and q_3, q_4, &c., are successively found from equations (2′), (3′), &c.

(*To be continued.*)

PROBLEMS.

1. Find the value of x and y in the following equations:

$$a^2 x^4 + b^2 y^4 = a^2 b^2 (x + y)^2;$$
$$a^2 x^2 + b^2 y^2 = a^2 b^2.$$

—Communicated by U. Jesse Knisely, Pres't and Prof. of Mathematics in *Luther College*, Newcomerstown, Ohio.

2. Let a regular polygon of 14 sides be described, each of whose equal sides shall be *one*. Then will the radius of its circumscribing circle, which put$=r$, be more than *two* and less than *three*. Put $r=2+x$; then is x a positive quantity less than *one*. Let another regular polygon of half the number of sides (7) be inscribed in a circle whose radius is *one*, and determine one of its equal sides in functions of x expressed in its simplest form.

3. If a line make an angle of 40° with a fixed plane, and a plane embracing this line be perpendicular to the fixed plane, how many degrees from its first position must the plane embracing the line revolve in order that it may make an angle of 45° with the fixed plane?—Communicated by Prof. A. Schuyler, Berea, Ohio.

4. A cask containing a gallons of wine stands on another containing a gallons of water; they are connected by a pipe, through which, when open, the wine can escape into the lower cask at the rate of c gallons per minute, and through a pipe in the lower cask the mixture can escape at the same rate; also, water can be let in through a pipe on the top of the upper cask at a like rate. If all the pipes be opened at the same instant, how much *wine* will be in the lower cask at the end of t minutes, supposing the fluids to mingle perfectly?—Communicated by Artemas Martin, Mathematical Editor of *Schoolday Magazine*, Erie, Pa.

Note.—To those who use "Nystrom's Mechanics:" Nystrom prints
"$\pi^2=9.869650000+$,"
but $\pi^2=9.86960440108+$.—U. Jesse Knisely.